Unit 11 Outline – Power Series

Monday 2/24	**Today's Topic:** Introduction to Power Series; Interval and Radius of Convergence

Key Idea:
If x is a variable, then the infinite series of the form:

$$\sum_{n=0}^{\infty} a_n (x-c)^n = a_0 + a_1 (x-c)^1 + a_2 (x-c)^2 + \cdots + a_n (x-c)^n + \cdots$$

is called a **Power Series** centered at $x = c$, where c is a constant.

For a power series centered at $x = c$, exactly one of the following is true:

1) The series converges at $x = c$ ONLY. (All power series converge at their center)
2) The series converges for all x.
3) The series converges on some interval $(c - R, c + R)$. R is called the **radius of convergence**.

In-Class Examples: Find the interval and radius of convergence for each power series:

1. $\sum x^n$
2. $\sum \frac{x^n}{n}$
3. $\sum \frac{x^n}{n!}$
4. $\sum n! x^n$
5. $\sum \frac{(2x-5)^n}{n^2}$
6. $\sum \frac{(-1)^n x^n}{3^n (n+1)}$

Homework: Worksheet 87

Tuesday 2/25	**Today's Topic:** Interval and Radius of Convergence

In-Class Examples: Find the interval and radius of convergence for each power series:

1. $\sum \frac{(-1)^{n+1} (x-5)^n}{n(2^n)}$
2. $\sum \left(\frac{x}{3}\right)^n$
3. $\sum \frac{(-1)^{n+1} x^{2n+1}}{(2n+1)!}$
4. $\sum n!(x-3)^n$

Homework: Worksheet 88

Wednesday 2/26	**Today's Topic:** To write the Taylor Series and Maclaurin Series approximations for given functions

Key Idea:

<u>Definition of an *n*th-degree Taylor polynomial:</u>

If f has n derivatives at $x = c$, then the polynomial

$$P_n(x) = f(c) + f'(c)(x-c) + \frac{f''(c)}{2!}(x-c)^2 + \cdots + \frac{f^{(n)}(c)}{n!}(x-c)^n$$

is called the <u>*n*th-degree Taylor polynomial for f centered at c</u>, named after Brook Taylor, an English mathematician.

Note 1: A first-degree Taylor polynomial is a tangent line to f at c.

Note 2: $\frac{f^{(n)}(c)}{n!}$ **is the coefficient of the** $(x-c)^n$ **term**

If $c = 0$, then $P_n(x) = f(0) + f'(0)(x) + \frac{f''(0)}{2!}x^2 + \cdots + \frac{f^{(n)}(0)}{n!}x^n$ is called the <u>*n*th-degree Maclaurin polynomial for f</u>, named after Scottish mathematician, Colin Maclaurin.

In-Class Examples: Write the Taylor series for the given functions

1) $f(x) = e^x, \quad c = 0$

2) $f(x) = \cos x, \quad c = \frac{\pi}{2}$

3) $f(x) = \frac{1}{x}, \quad c = 2$

Homework: Worksheet 89

Thursday 2/27	**Today's Topic:** More work on Taylor/Maclaurin Series

Key Ideas: MEMORIZE!!!

TABLE 1

Important Maclaurin Series and Their Radii of Convergence

$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \cdots$	$R = 1$
$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$	$R = \infty$
$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$	$R = \infty$
$\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$	$R = \infty$
$\tan^{-1} x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots$	$R = 1$
$(1+x)^k = \sum_{n=0}^{\infty} \binom{k}{n} x^n = 1 + kx + \frac{k(k-1)}{2!}x^2 + \frac{k(k-1)(k-2)}{3!}x^3 + \cdots$	$R = 1$

Homework: Worksheet 90

Friday 2/28	**Today's Topic: Review**
In-Class Examples: None	
Homework: Worksheet 91	

Monday 3/4	**Today's Topic: Review**
In-Class Examples: None	
Homework: Worksheet 92	

Tuesday 3/5	**Today's Topic: Review**
In-Class Examples: None	
Homework: Worksheet 93	

Wednesday 3/6	**Today's Topic:** Exam
In-Class Examples: None	
Homework: None	

Thursday 3/7	**Today's Topic:** Free Response Questions
In-Class Examples: None	
Homework: Worksheet 94	